local. In such case, the triggering results and conclusions drawn from a two-mode analysis would probably be meaningless. However, the Comment does not explain how the parametric results for variation in α_1 justify such a global conclusion.

Conclusion

It is recognized that the problem of triggered instabilities is a complicated one and that it is difficult to analyze the problem from a comprehensive vantage point while at the same time seeking both physical understanding and quantitative accuracy. As such, it is believed that Refs. 1 and 2 represent two studies that each contribute significant progress in researching nonlinear instabilities. As this is a difficult problem involving many uncertainties, both individual methods have their own weakness and merit.

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Comment on "Shock-Loss Model for Transonic and Supersonic Axial Compressors with Curved Blades"

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A RECENT paper[1] addresses the difficult flow at the entry region to supersonic compressor blades. There are serious errors in some of what is written by Schobeiri about a paper we wrote a few years ago[2] and a flaw in the model he describes in his paper.[1]

We produced an approximate method for the calculation of the stagnation pressure loss and static pressure rise in the inlet region of blades with supersonic inlet velocities. The approximations are appropriate for the type of blades that we have in mind, examples

Received 1 October 1998; accepted for publication 11 March 1999.

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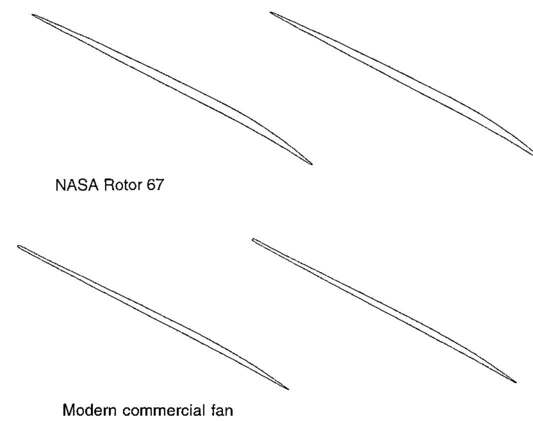

Fig. 1 Two sections through rotors at 90% span.

of which are shown in Fig. 1. Both the blades shown are at the 90% span location; one is the NASA Rotor 67, the other is for a modern high-performance fan for a commercial aircraft. Unlike the blades shown in Schobeiri's paper, the blades are both thin and nearly flat over the forward part. Because of the thinness and flatness, it is a good approximation in the forward part to neglect the force produced by the blades along their direction in computing the pressure and momentum balance in this direction; a momentum balance in any other direction would require the blade force to be explicitly included. Neglecting the thickness of the blades in the momentum equation, particularly at the leading edge, is only an approximation, but as Denton has shown (private communication, 1998), including this has only a very small effect. The thickness of the blades is not, however, neglected in the mass continuity equation because around $M = 1.0$ the flow is exceptionally sensitive to small variations in flow area.

As authors of Ref. 2, we must accept responsibility for Schobeiri misunderstanding what we were doing. It is, however, unfortunate that he has represented his misunderstanding in terms of our ineptitude. For example, he states that considerable discrepancies existed between the left- and right-hand sides of the momentum equation; this is simply untrue. What we did was to plot the left- and right-hand sides of the equation to show that equality could only occur for particular combinations of inlet Mach number M_1 and Mach number M_2 at outlet from the inlet region. Other criticisms Schobeiri makes are either untrue or are part of the approximation that we have reason to believe is realistic.

Before moving on from Schobeiri's misconceptions about Ref. 2, it is worth mentioning that the goal of a simple model like this is to give insight into the main controlling factors or parameters. The requirement is no longer to come up with the most complete description of the flow; now we have three-dimensional computational fluid dynamics (CFD) to do this for us. (In this respect the need is quite different from, for example, that in the 1950s when people such as Levine[3] were trying to give the most complete and accurate description possible for the leading-edge region in supersonic flow.) Simple methods, which isolate a few effects, are useful if they can give us understanding that the more complete description does not. Thus, for example, Ref. 2 showed that when a strong shock is ahead of the leading edge the minimum loss in two dimensions is that of the normal shock, but the loss can be a lot higher if the blade is thick and the incidence increases. The principal weakness of any such method based on two dimensionality is the need to include the variation in streamtube thickness in the spanwise direction. Even small changes can have a very pronounced effect on both pressure rise and loss. The only realistic way to determine these streamtube thickness variations is with three-dimensional CFD.

The flows in many fan geometries have now been calculated by three-dimensional Reynolds-averaged Navier–Stokes methods. Two examples for NASA Rotor 67 are shown in Fig. 2, where contours of Mach number computed using the Denton TIP3D code are shown at the 90% span location. In both cases shown, the rotational speed is the design value, but one case is near peak efficiency and the

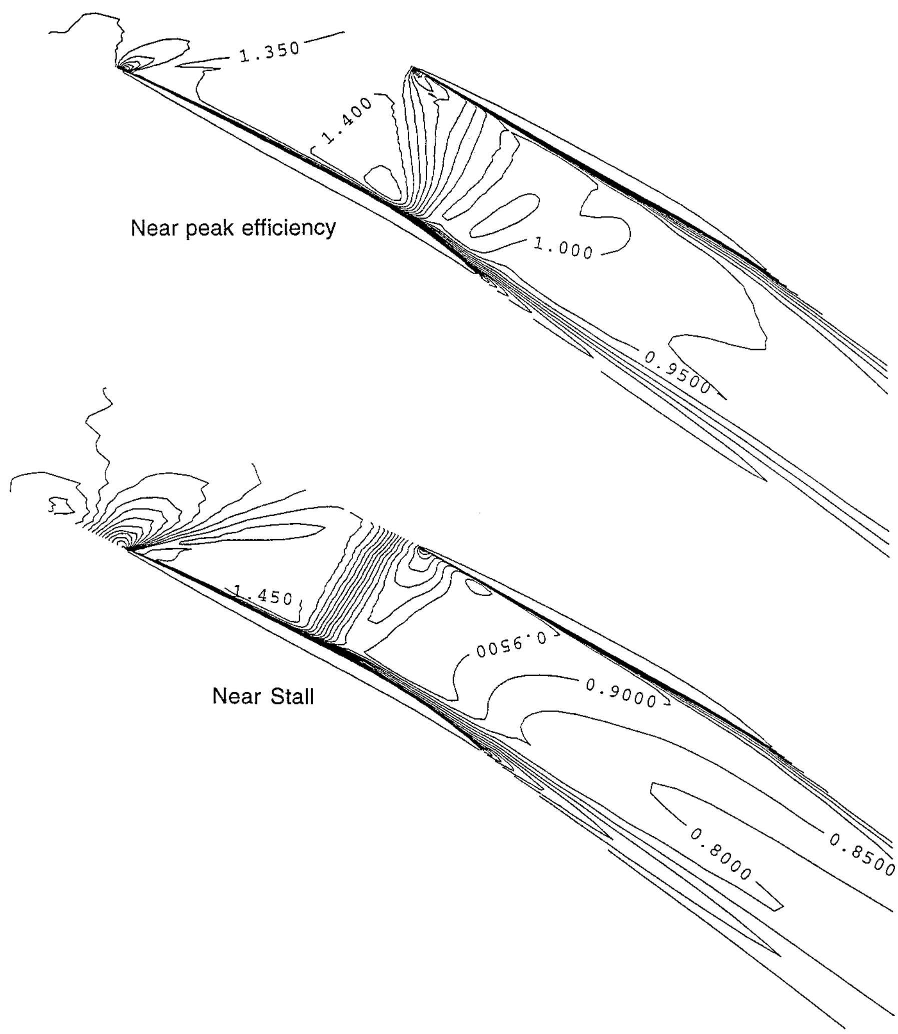

Fig. 2 Contours of relative Mach number for NASA Rotors 67 computed at design rotational speed; section shown is at 90% span.

other is near to stall. Even near peak efficiency, the bow shock is slightly ahead of the leading edge; the unique incidence condition corresponds to choking and is not the condition at which highest efficiency is normally found, nor is it the condition at which the blades normally operate. In both cases shown, it can be seen that it is a remarkably good approximation to take the Mach number to be uniform in the pitchwise direction downstream of the shock; for the peak efficiency case, it is more precise to say downstream of the region of shocks. If the Mach number is uniform, so too is the static pressure, as was assumed in the Freeman and Cumpsty model.[2]

The model for supersonic inlet behavior described by Schobeiri[1] seems to contain a serious flaw. He decided to evaluate the balance of momentum and pressure forces in the tangential direction. This requires the tangential force to be obtained from the integrated pressure distribution along the blade and along any other surface of the control volume that is not parallel to the tangential direction unless periodicity causes the force component to cancel. (It was to avoid the need to know the force on the blade that the component of momentum parallel to the nearly flat blade surface was used by Freeman and Cumpsty.) Schobeiri states that the pressure will be equal at points A and C in his Fig. 2, points that are on either side of the leading edge. This is not a good assumption because an expansion around the leading edge is commonly encountered. Furthermore, the curvature along the suction surface that he assumes, and that is an essential part of his model, leads to a fan of expansion waves. As a result there is no reason why the integrated pressure force in the tangential direction along the blade suction surface upstream of the shock (BC in Schobeiri's Fig. 2) should be even approximately equal to the tangential force along the shock (AB in Ref. 1). This, therefore, affects the solution of the momentum equation [Eq. (6)] in Schobeiri's paper.

In Fig. 2 of this Note, the Mach number contours for Rotor 67 show the shock forward of the leading edge, just slightly for peak efficiency and well forward near stall. In this configuration it is clear that the pressure force along the leading edge in the tangential direction cannot balance the tangential force along the shock, as assumed in the model presented by Schobeiri.

Acknowledgment

The authors would like to thank Liping Xu for his help by carrying out the computational fluid dynamics analysis, the result of which is shown in Fig. 2. The help of John Denton in this and in the underlying modeling of the flow is also gratefully acknowledged.

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